Quantitative Network Design: What is it and what can we do with it?

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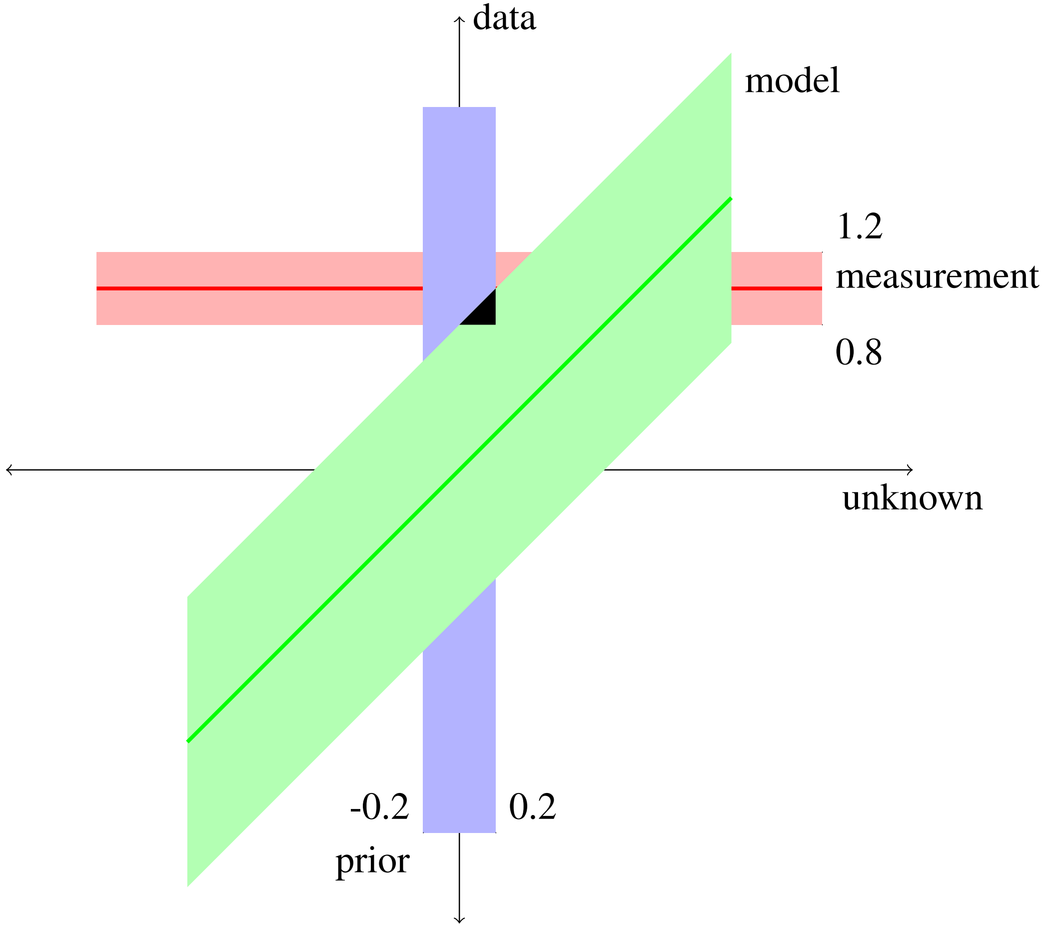
Outline

- How to combine data and models;
- What is Quantitative Network Design (QND)?
- The question determines the answer;
- More complicated models, tracking uncertainty around;
- Expert judgment, evaluation and optimisation.

Data Assimilation in One Picture



- Unknown on X-axis, obs on Y-axis;
- Light-blue = prior unknown
- Light-red = obs
- Green = model;
- Black = solution.

Notes

- Location of triangle indicates value of solution;
- Size of triangle indicates uncertainty of solution;
- Uncertainty (size) often independent of value (location);
- Hence can often calculate uncertainties *without* real measurements.

A tiny bit of Theory

- Information additive so

$$\text{PosteriorInfo} = \text{PriorInfo} + \text{InfoFromMeasurements}$$

$$\text{information} = 1/\text{uncertainty}$$

- Information passed from measurements to unknowns via model:

$$\mathbf{C}(\vec{x}) = \mathbf{J}^T \mathbf{C}(\vec{d})\mathbf{J}$$

where $\mathbf{C}$ is uncertainty (covariance) and $\mathbf{J}$ sensitivity of measurements $\vec{d}$ to unknowns $\vec{x}$.

Propagating Uncertainty through Model

- Can also calculate uncertainty in a different quantity $\vec{y}$

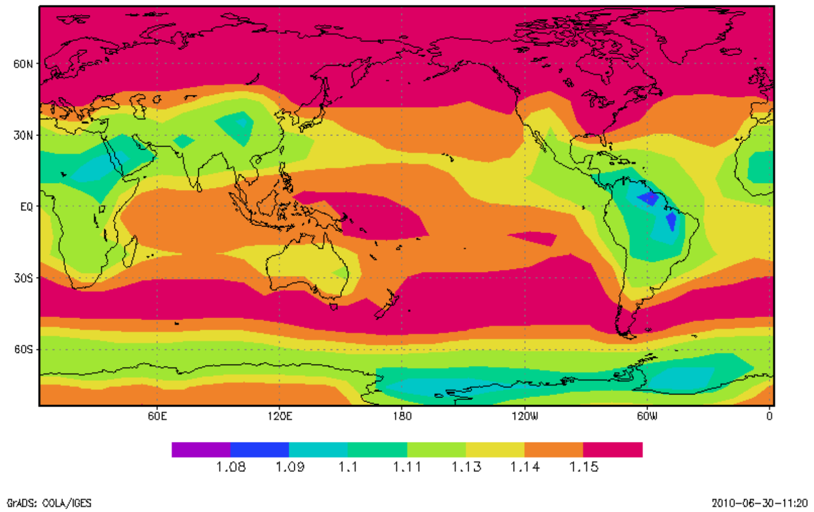
$$\mathbf{C}(\vec{y}) = \mathbf{J}\mathbf{C}(\vec{x})\mathbf{J}^T$$

- Can combine to calculate uncertainty in $\vec{y}$ after measurements.

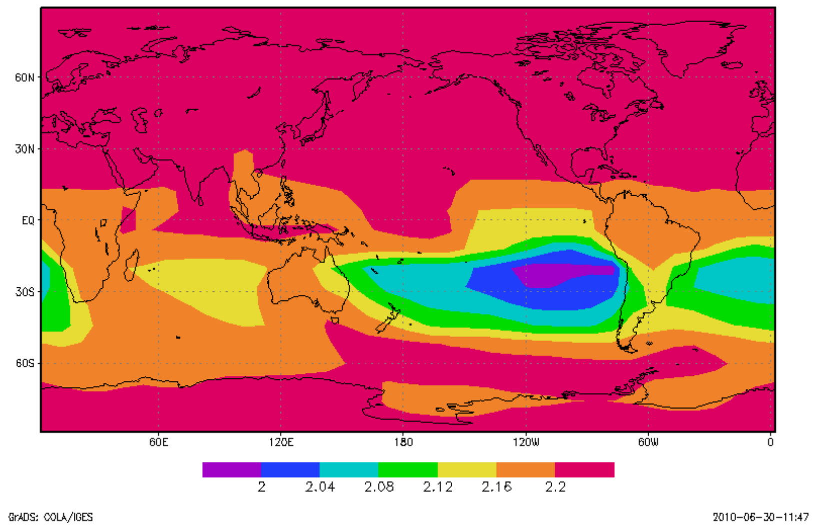
Optimisation, Evaluation or Expert Judgment?

- Can *evaluate* or even map $\mathbf{C}(\vec{y})$ for a given network;
- Can use algorithm to *optimize* some aspect of $\mathbf{C}(\vec{y})$;
- Optimization useful since it can surprise you;
- Any evaluation or optimisation misses much practical knowledge so finally expert judgment is the arbiter.

For Optimization, the answer depends on the question

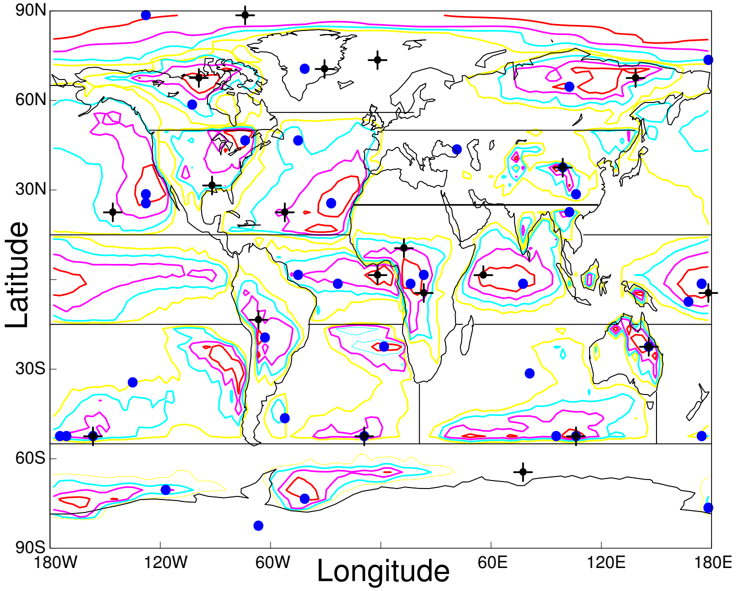


$$\mathrm{var}\left(\sum_1^N f_{O,i}\right)$$

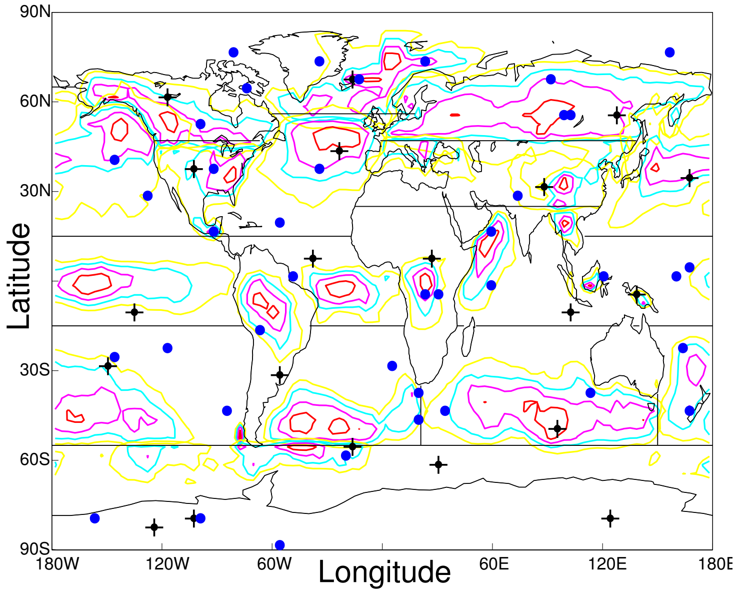


$$\sum_1^N \mathrm{var}(f_{O,i})$$

Comparing Error Models



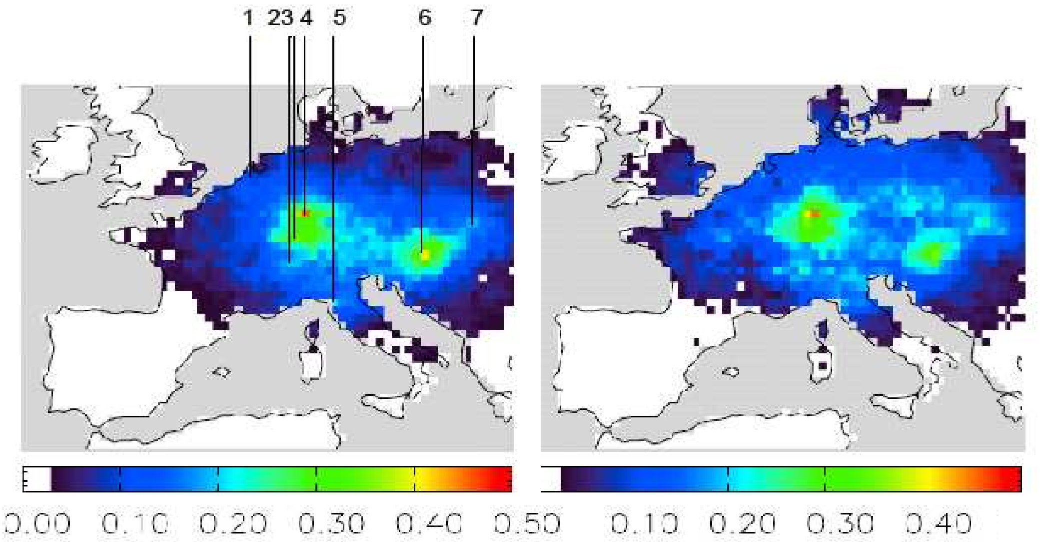
Optimal network in the case where all data uncertainties are equal



Optimal network in the case where data uncertainties vary according to simulated variability

Using Higher Frequency Data

Nouvelle réduction d'erreur sur les flux , moyenne de l'été 2003



D'Ouest en Est :

1-CBW, 200m

2-PRS, 3485m

3-SCH, 1205m

4-HEI, 116m

5-CMN, 2165m

6-HUN, 363m

7-KAS, 1987m

TEMOIN : Erreur indexée sur la respiration du modèle ORCHIDE [Chevallier et al., 2006]

SECOND : Erreur indexée sur la dev. std. entre 10 modèles de flux [Rodenbeck et al., 2003]

Céline Aulagnier PhD thesis. Reduction of error on 6-hourly fluxes using hourly data with network as of 2003

Network Design in a Data Assimilation Context

- CCDAS combines different kinds of observations;
- Must consider representativeness of measurements along with all previous caveats;
- Network Optimization will be model dependent;
- More complex optimization since we can trade off different measurement types as well as locations.

Lessons

1. Define your question carefully (multi-criterion optimization helps);
2. Optimized network depends on prior knowledge;
3. Optimized network depends on error model;
4. Also depends on observation operator;
5. Different kinds of uncertainty will generate different networks;
6. Can work in a CCDAS context but model dependent.

Random Thoughts

- There are many more network design issues we don't seem to be considering here: fossil fuel, tower height, interplay of satellite and surface measurements;
- Signal detection may require different kinds of networks;
- We need to guess at model advances.